Relevance of the Type III error in epidemiological maps

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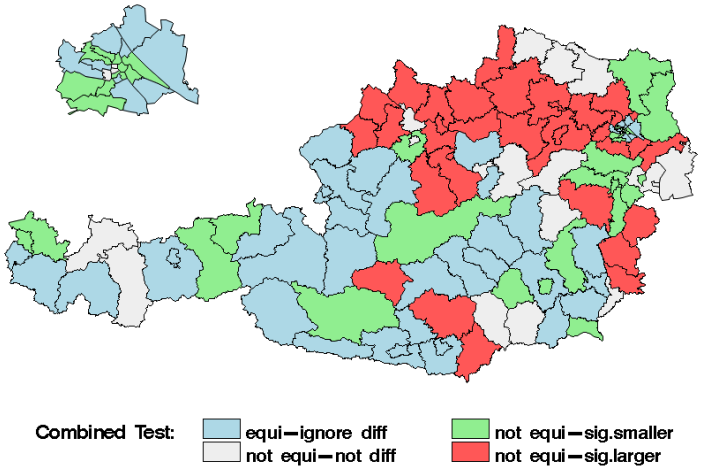
Test for difference in maps

Two-sided test

H0: SMR=1 vs. HA: SMR≠1

Respiratory diseases



Combined test

H0: SMR=1 vs. HA: SMR≠1

How „trustful“ is an observed significant test result?

Errors in classical decision making

Type I - believe in alternative hypothesis though null hypothesis is true

Type II - believe in null hypothesis though alternative hypothesis is true

What is worse than Type I and II error?

Effect reversal

Observe a significant risk in one direction

but true risk is the other way round

Example effect reversal

In a district the true unknown SMR = 1.2

expected cases under H0 = 6

observed cases =1

Crude $SMR = \frac{\text{observed}}{\text{expected}} = \frac{1}{6} = 0.16$ (95% CI: 0.01-0.93)

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Crude $SMR = \frac{\text{observed}}{\text{expected}} = \frac{1}{6} = 0.16$ (95% CI: 0.01-0.93)

We observe significantly decreased risk of .16 ->

we believe that true risk is <1 though it is >1

Observing a significant result in one direction

though true effect is in the other direction

Type III error

Kaiser [1]

ticularly repugnant γ_{13} and γ_{31} errors —"errors of the third kind"—have

1) "Directional Statistical Decisions", *Psychological Review*, 67 (3), 1960

Directional tests

H1: SMR<1 H2: SMR=1 H3: SMR>1

Test Decision	True Nature			
		H1 SMR<1	H2 SMR=1	H3 SMR>1
	H1 SMR<1	Correct	α	γ
	H2 SMR=1	β	Correct	β
	H3 SMR>1	γ	α	Correct

Implications of γ and γ may be different

q – value

What is the probability obtaining a wrong-sided significant result if the observed result is significant?

Heinzl* H, Benner A, Ittrich C, Mittlböck M (2007). Proposals for Sample Size Calculation Programs. *European MethodsInfMed;46:655–661*.

For crude SMRs

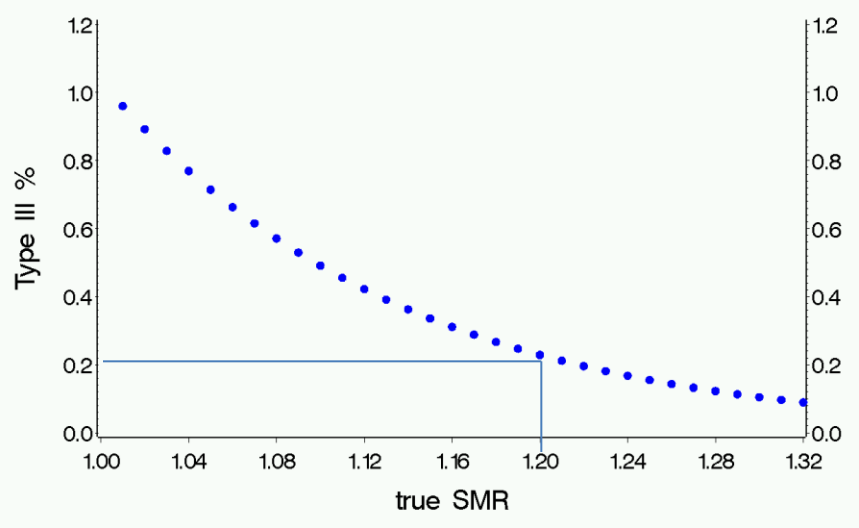
Type III and q-value

may be calculated analytically

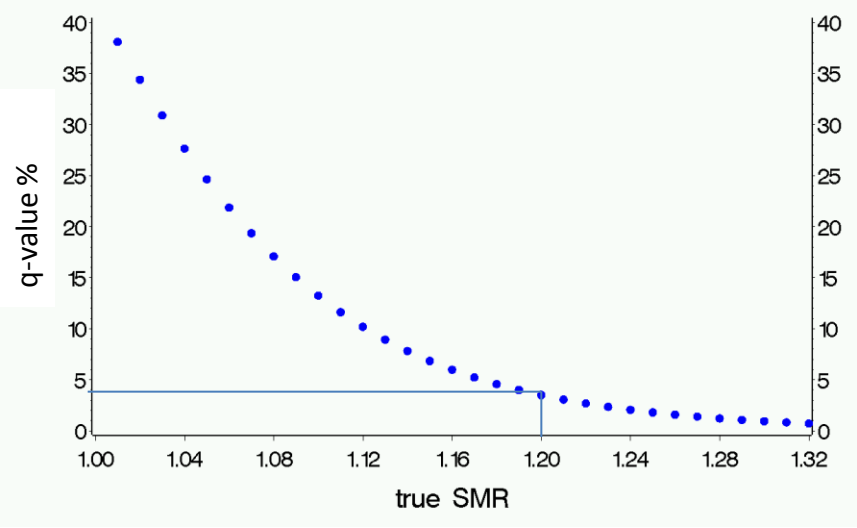
(crude SMR hardly used in spatial epidemiology)

Type III error and q-value against true SMR for # expected cases = 10

Type III error



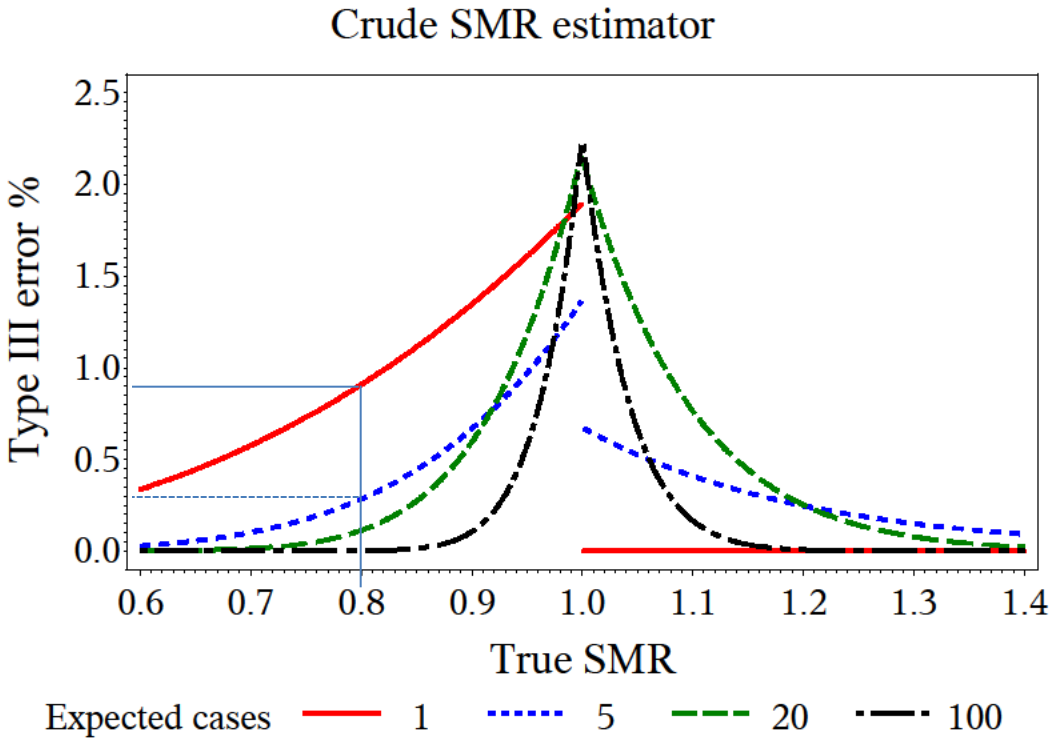
q-value



True SMR=1.2: Type III = 0.2%

q-value ~ 4%

Type III error dependent on true SMR and number of expected cases



Heinzl H, Waldhoer T, Relevance of the type III error in epidemiological maps, IJHG 2012, 11:34
http://www.ij-healthgeographics.com/content/11/1/34

In Spatial Epidemiology

Random effect (RE) models often used

Spatially

Unstructured and/or *Structured model*

(BYM)

Besag J, York J, Mollié A: **Bayesian image restoration, with two applications in spatial statistics (with discussion).** Annals of the Institute of Statistical Mathematics 1991, **43**(1):1-59.

Spatially

unstructured RE models shrink to a *global* mean

(e.g. mean of Austria)

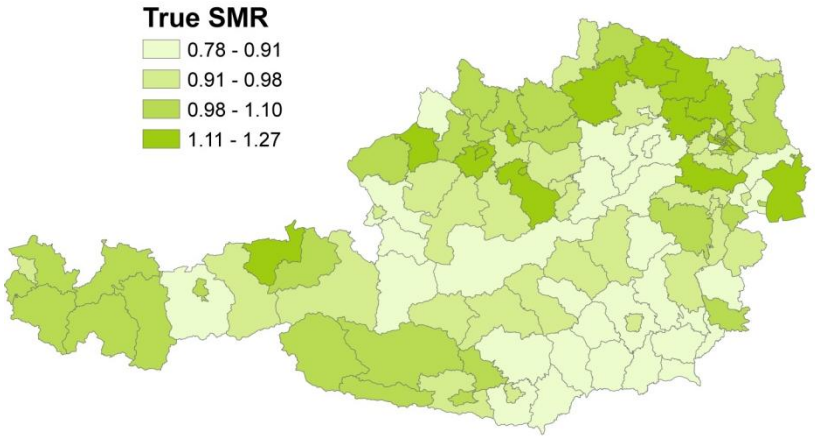
structured RE models shrink to a *local* mean

(e.g. mean of neighbours)

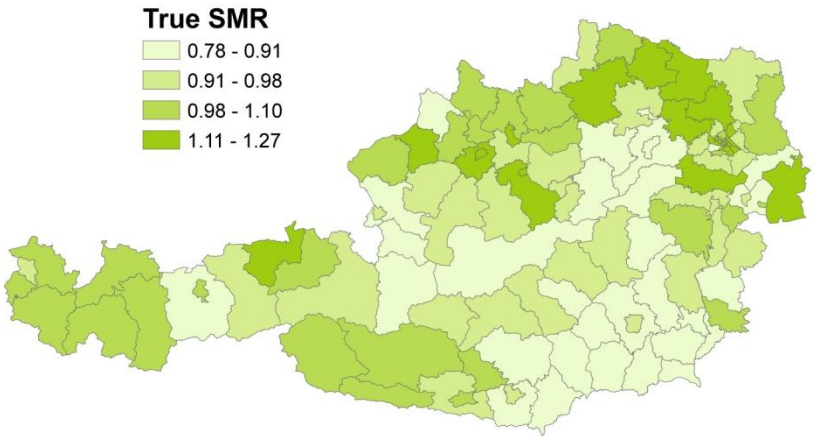
Our question

What is the effect of shrinkage of spatially structured and unstructured RE models in respect with Type III error and q-value ?

Simulation of infant mortality data based on a predefined spatial risk



Simulation of infant mortality data based on a predefined spatial risk



1) Model estimation of SMR in INLA, R

2) Calculation of Type III and q-values using a decision rule based on the posterior distribution

Decision rule for being „*significant*“

Posterior distribution f(Δ/data)

Reference treshold Δ (e.g. 1)

Cutoff prob ω_1, ω_2 (e.g. 0.8)

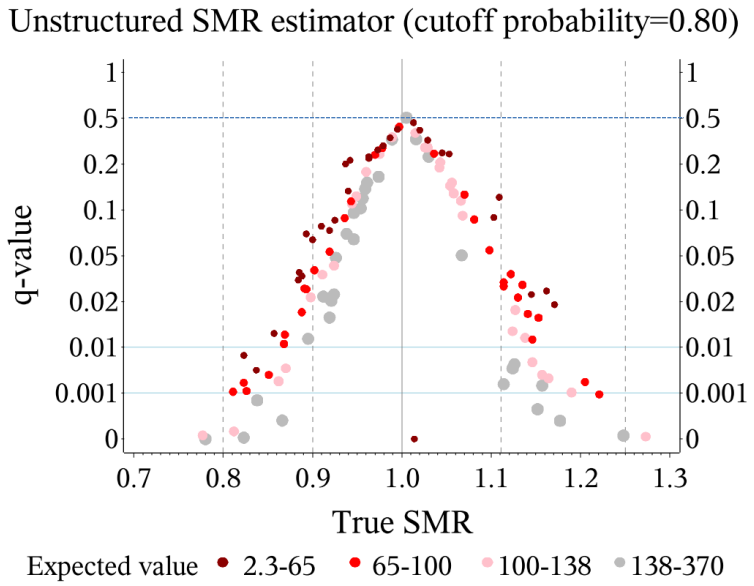
Two-sided „significant“ decision rule:

$P(\Delta>\Delta_{01})> \omega_1$, $P(\Delta<\Delta_{02})> \omega_2$

$P(\Delta>1) > 0.8$, $P(\Delta<1) > 0.8$

Richardson S, Thomson A, Best N, Elliott P: **Interpreting posterior relative risk estimates in disease-mapping studies.** Environmental Health Perspectives 2004, **112**(9): 1016–1025.

Results for simulated infant mortality data for q-value for spatially **unstructured** and **structured** models (ω=0.8) in dependence on SMR and expected cases



unstructured model
Shrinkage to mean
of Austria

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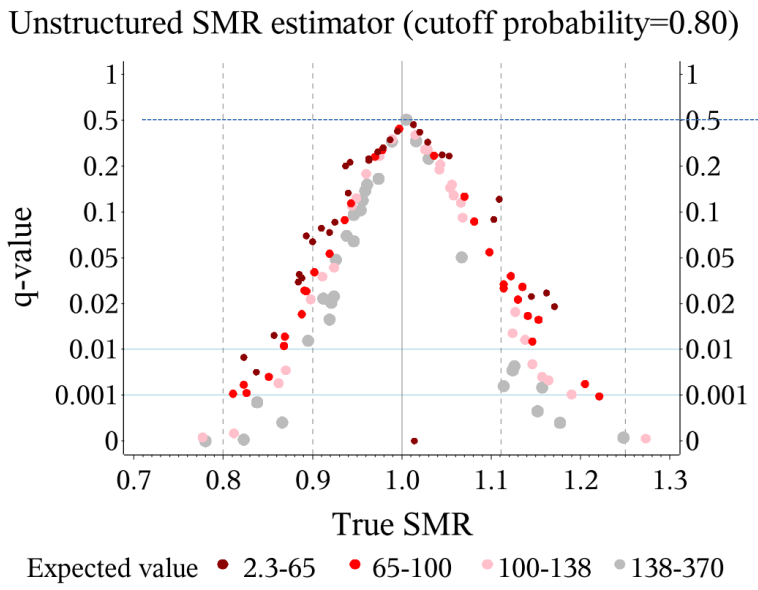
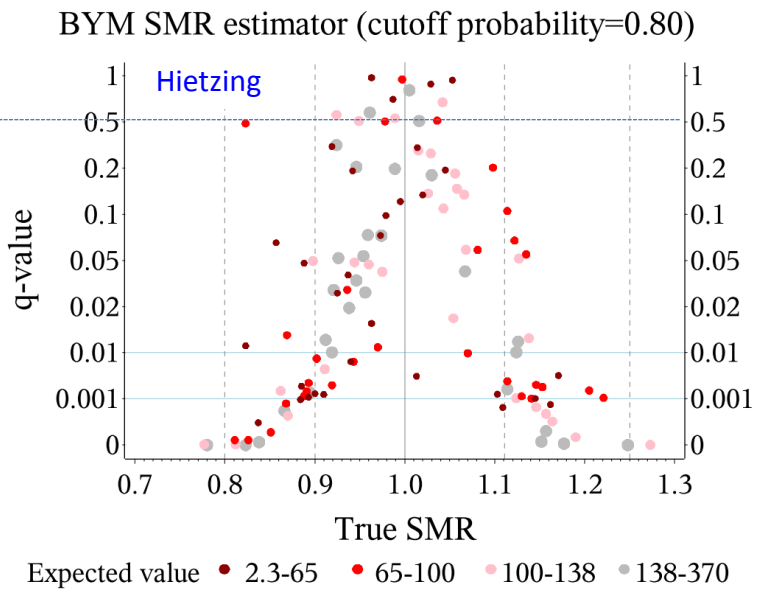
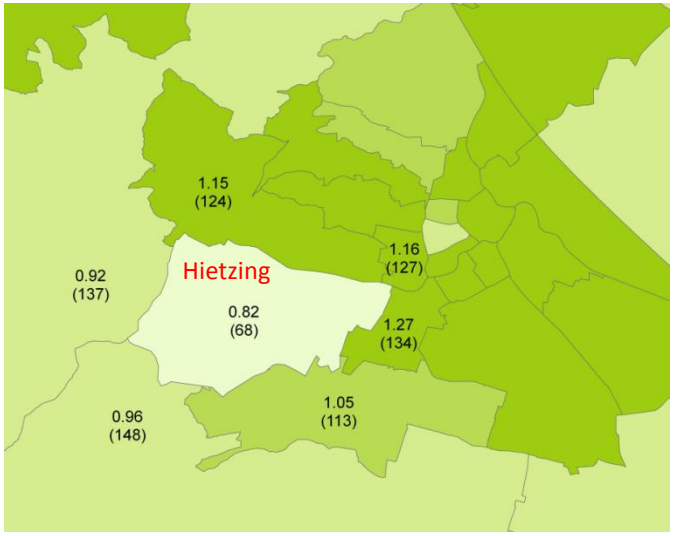




unstructured model
Shrinkage to mean
of Austria

structured model
Shrinkage to mean
of neighbours

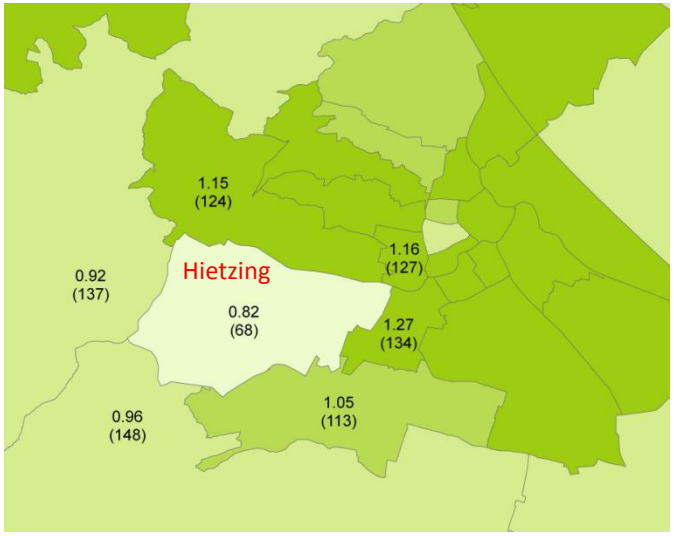
Effect reversal of estimated SMR for district Hietzing due to neighbours with larger SMRs



Parameters in simulation for the 7 districts

	Hietzing	neighbours
true SMRs:	0.82, 0.92, 0.96,	1.05, 1.15, 1.16, 1.27
Expected cases:	68, 113, 124,	127, 134, 137, 148

Effect reversal of estimated SMR for Hietzing due to neighbours with larger SMRs



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Expected cases:	68, 113, 124,	127, 134, 137, 148
Type III:	13%	
q-value:	50% = 13%/26%	
non-directional Power:	26%	

Conclusion

Be aware that „significant“ effects may be due to **effect reversal**

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For small SMRs and small number of expected cases Type III error and q-value may be relevant