# Spatio-temporal Modeling of Roe Deer Hunts

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# Goals

1. Detect spatio-temporal trends in roe deer hunting.

# Data

Roe deer hunts:

- daily records of hunts
- in 42 administrative districts of the Baden-Württemberg state forest service
- for the hunting seasons 2006,. . . ,2010 (May 1 – January 31)

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# Methods

We observe the hunts $y_{ijk}$, representing count data,

- on day $k = 1, \ldots, 276$
- of season $j = 2006, \ldots, 2010$
- in discrete region $i = 1, \ldots, 42$.

We assume

$$y_{ijk} \sim \text{Poisson}\left(\lambda_{ijk}\right) \ .$$

Hereof, the mean

$$\lambda_{ijk} = A_i \pi_{ijk}$$

is the product of the constant area $A_i$ of region $i$ and the rate $\pi_{ijk}$.

# Methods

Assume a latent Gaussian model with linear predictor on the logarithmic scale

$$\eta_{ijk} = \log\left(\pi_{ijk}\right) = \beta_0 + \sum_{m=1}^{M} \beta_m z_{ijk}^{(m)} + \sum_{l=1}^{L} f_l\left(u_{ijk}^{(l)}\right),$$

with

- a vector of unknown fixed coefficients $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_M)'$ for the covariates $\boldsymbol{z} = \left(x^{(1)}, \ldots, x^{(M)}\right)'$
- a collection of unknown functions $\boldsymbol{f} = \{f_1(\cdot), \ldots, f_L(\cdot)\}$ for the covariates $\boldsymbol{u} = \left(u^{(1)}, \ldots, u^{(L)}\right)'$

# Methods

**Assume**

- $\boldsymbol{x}$ to be a latent Gaussian field holding $\boldsymbol{\eta}$, $\boldsymbol{\beta}$, $\{f_l\}$
- it's density $\pi\left(\boldsymbol{x}|\boldsymbol{\theta}_1\right)$ to be Gaussian with zero mean and precision matrix $\boldsymbol{Q}\left(\boldsymbol{\theta}_1\right)$ with hyperparameters $\boldsymbol{\theta}_1$
- $\pi\left(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{\theta}_2\right)$ as distribution of the observable response variable

**Goal of Bayes inference:**
to compute posterior marginals of the Gaussian variables

$$\pi\left(x_i|\boldsymbol{y}\right) = \int \pi\left(x_i|\boldsymbol{\theta}, \boldsymbol{y}\right) \pi\left(\boldsymbol{\theta}|\boldsymbol{y}\right) \mathrm{d}\boldsymbol{\theta}$$

as well as of the hyper-parameters

$$\pi\left(\theta_j|\boldsymbol{y}\right) = \int \pi\left(\boldsymbol{\theta}|\boldsymbol{y}\right) \mathrm{d}\boldsymbol{\theta}_{-j}$$

# Methods

The INLA (integrated nested Laplace approximations) method is here used as computationally cheaper alternative to MCMC to obtain the analytically intractable posterior marginals.

# Methods

The linear predictor is

$$\begin{aligned}\eta_{ijk} =& \beta_0 + \beta_1 \cdot \texttt{prec}_{ijk} + \beta_2 \cdot \texttt{temp}_{ijk} + \beta_3 \cdot \texttt{sprucedec}_i \\ &+ f_{season}\left(k\right) + f_{week}\left(g(jk)\right) + \omega_j + \upsilon_i\end{aligned}$$

with

- $\texttt{prec}_{ijk}$: median precipitation on day $k$ of season $j$ in region $i$
- $\texttt{temp}_{ijk}$: temperature anomaly
- $\texttt{sprucedec}_i$: decrease of spruce [%] from NFI 2 to NFI 3
- $f_{season}\left(k\right)$: nonparametric seasonal effect for the hunting season
- $f_{week}\left(g(jk)\right)$: nested seasonal effect for the calendar week; here $g(jk) \in \{1, \ldots, 7\}$ yields the weekday
- $\omega_j$: iid random error for the hunting season
- $\upsilon_i$: spatially structured error

# Methods

The spatially structured error builds a Markov Random Field (MRF) on discrete locations (regions).
It is modeled using a conditional autoregressive structure according to the Besag-York-Mollié specification

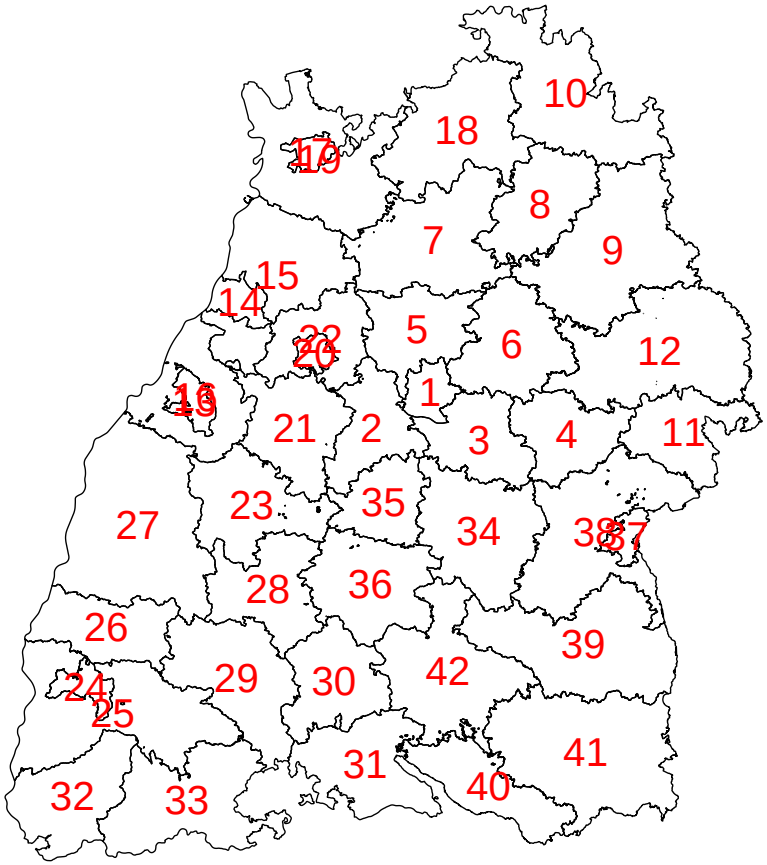
$$v_i \mid v_{i \neq j} \sim \text{Normal}\left(m_i, s_i^2\right) \text{ , with}$$

$$m_i = \frac{\sum_{j \in \mathcal{N}(i)} v_j}{\#\mathcal{N}(i)} \quad \text{and} \quad s_i = \frac{\tau_v^{-2}}{\#\mathcal{N}(i)}$$

Vague log Gamma priors are used for the spatial effect

$$\log \tau_v = \log \Gamma\left(1, 0.0005\right)$$

# Methods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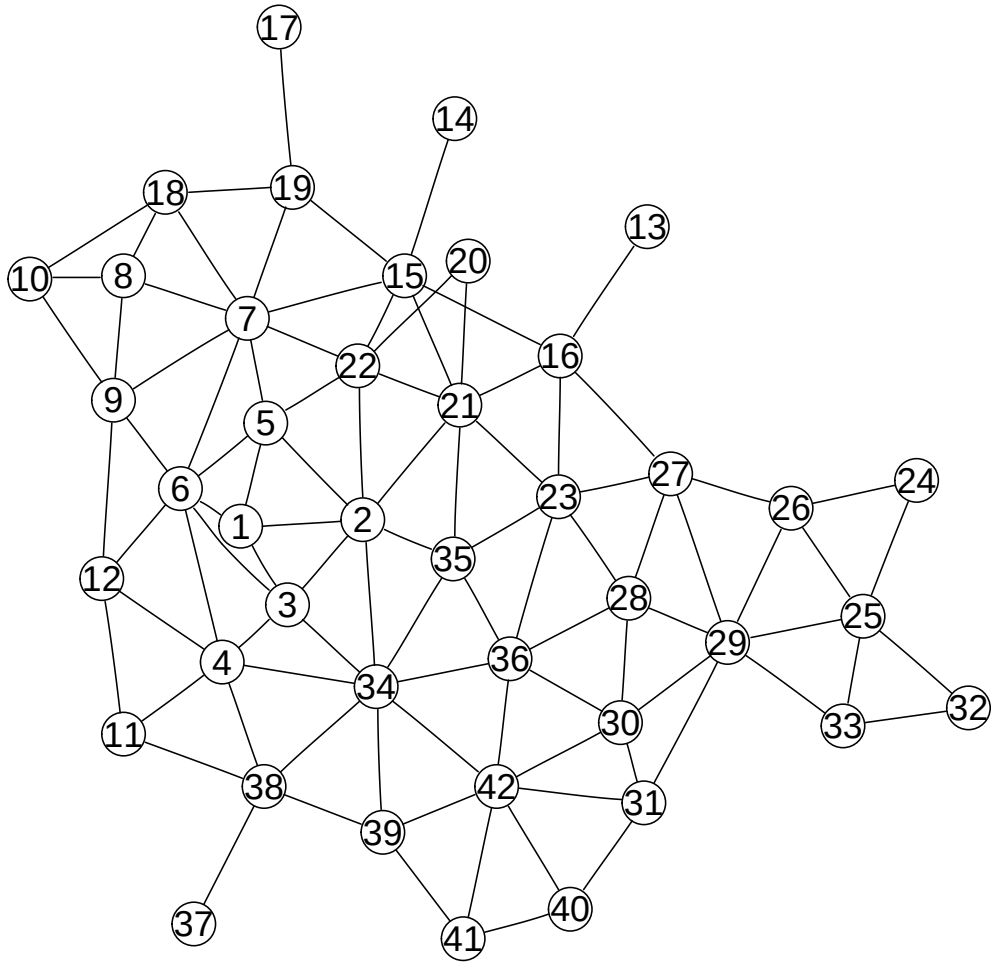
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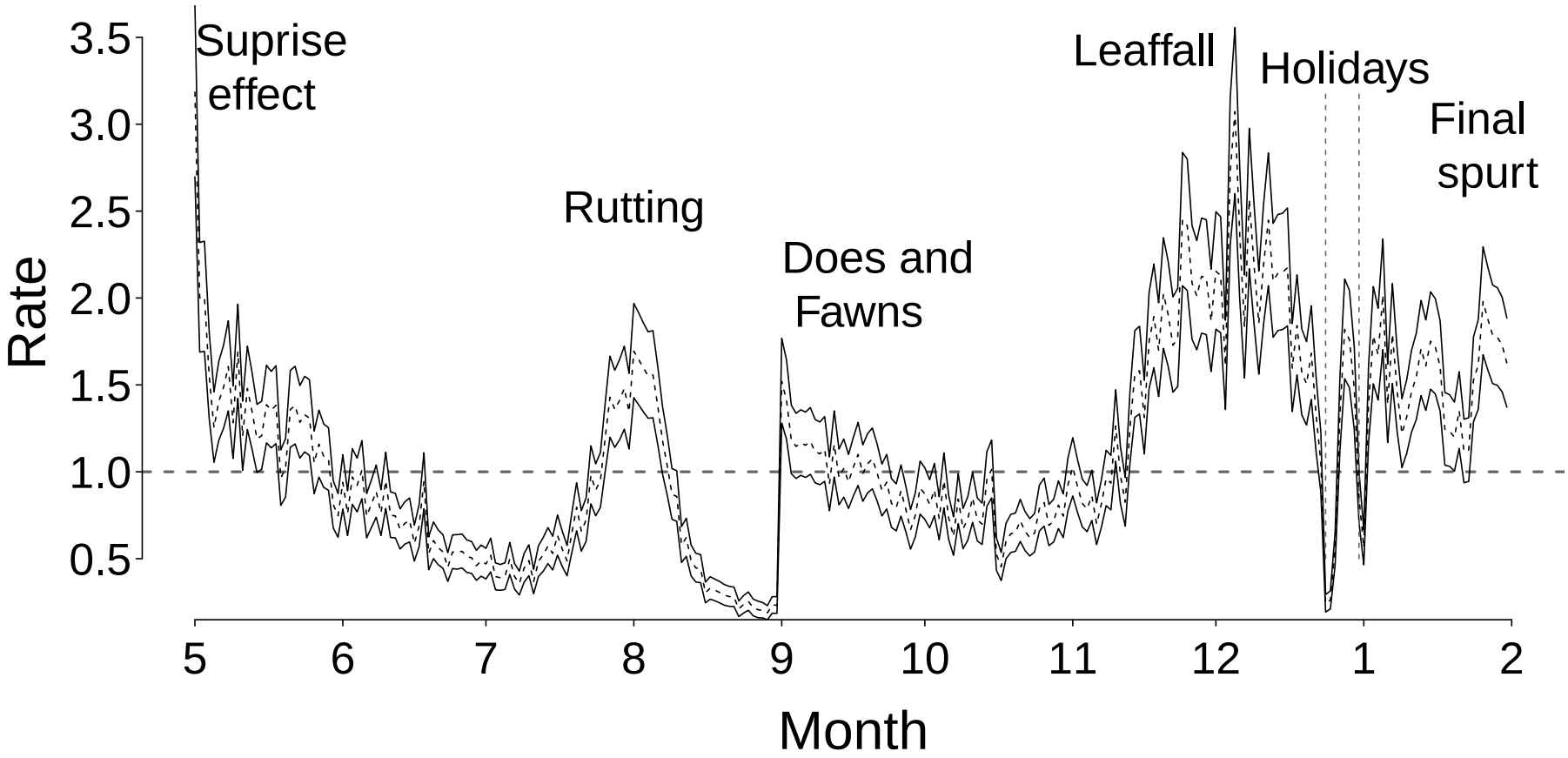
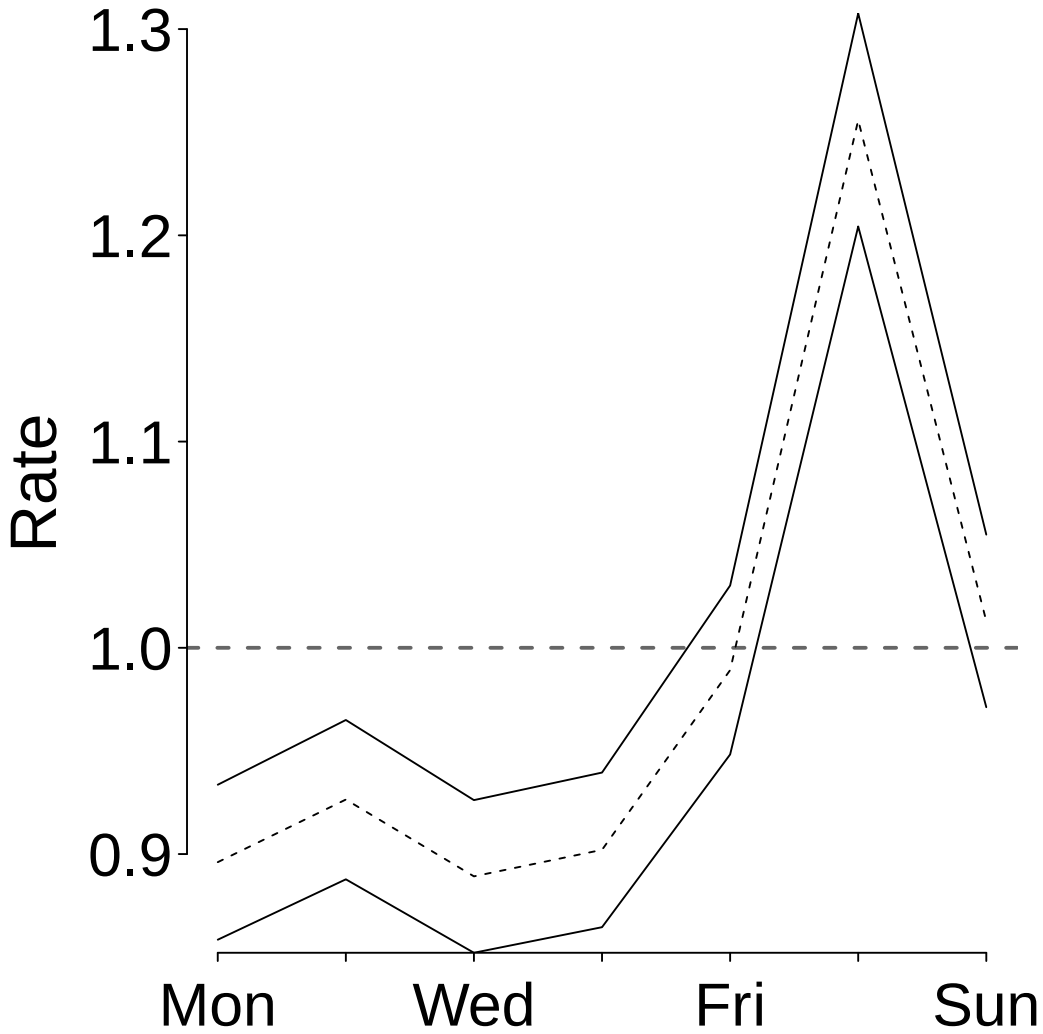
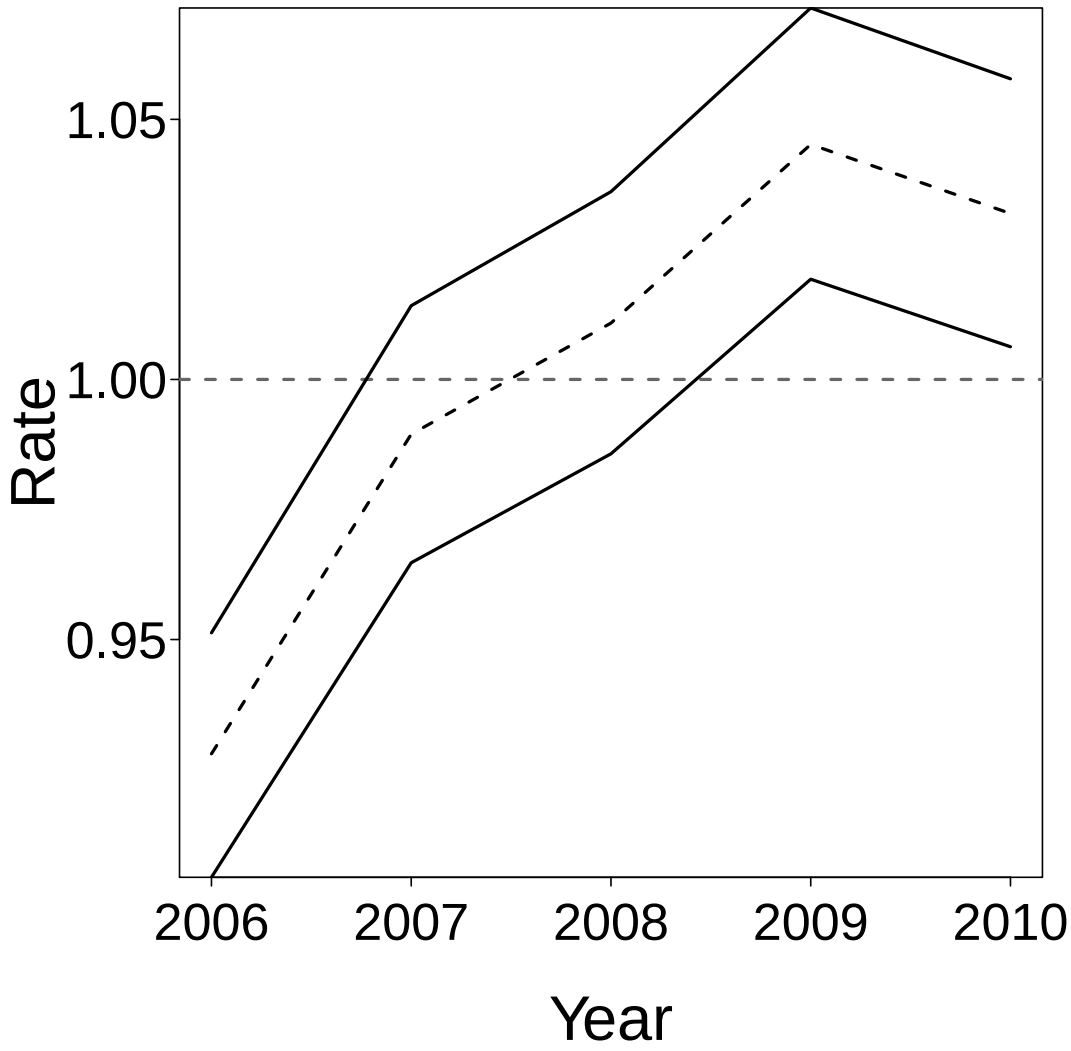
# Seasonal trend – Hunting season



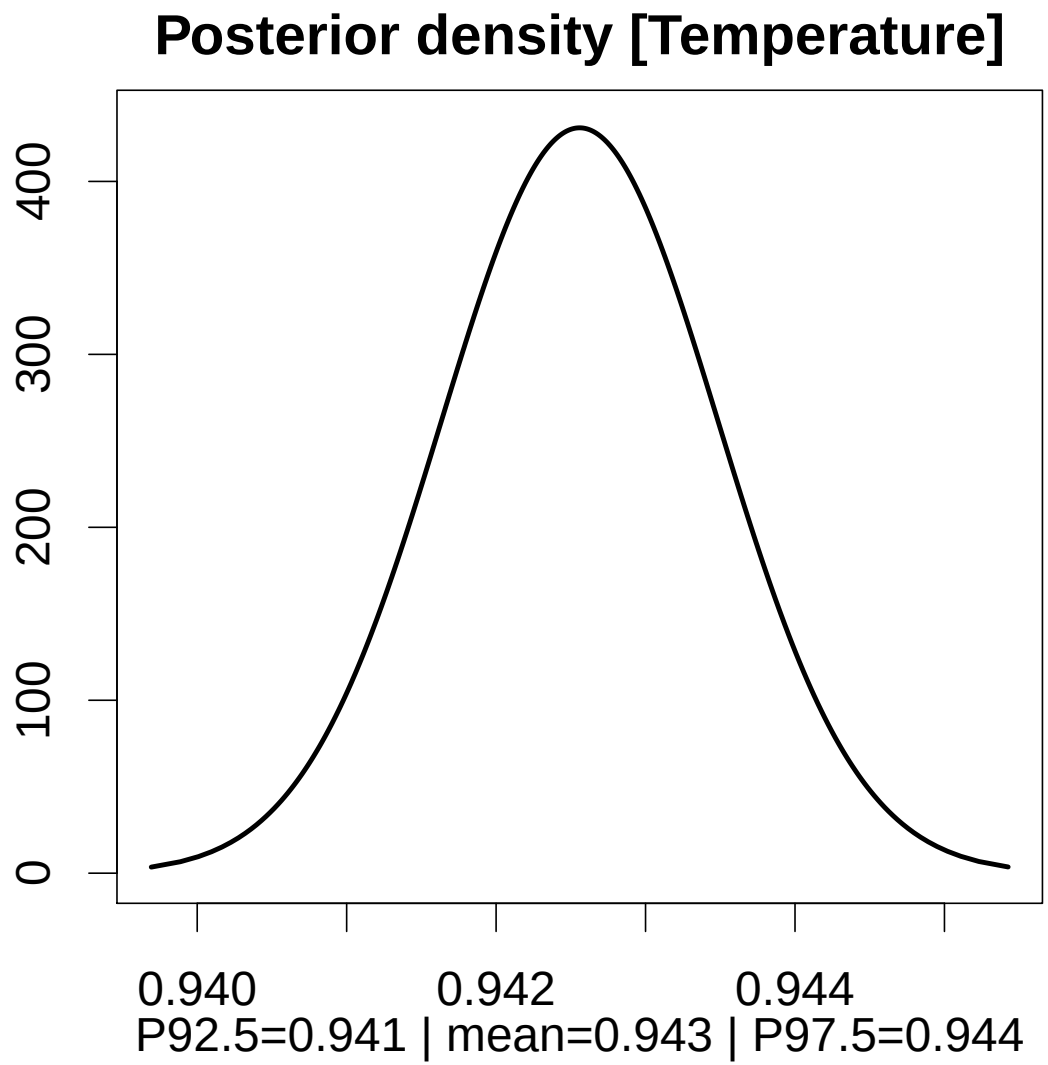
# Seasonal trend – Day of the week



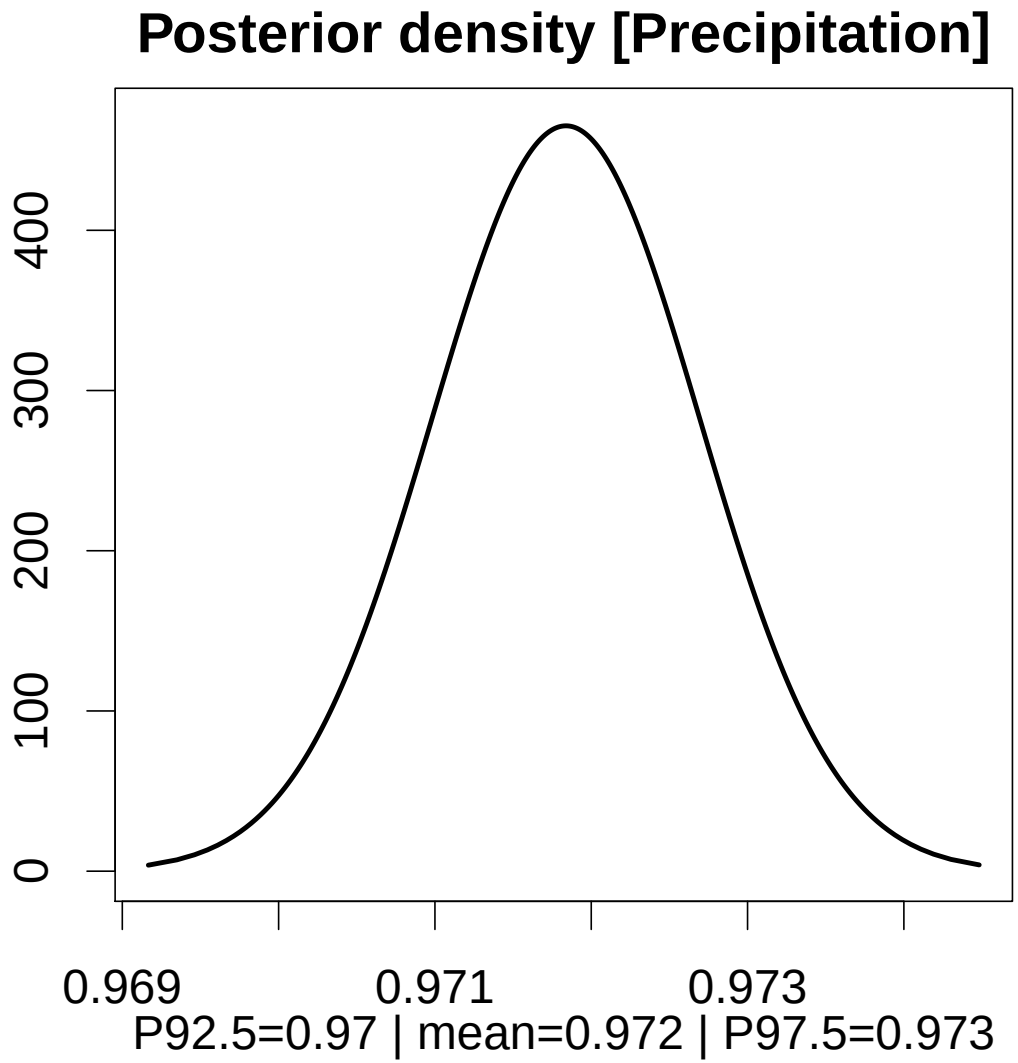
# Random error for hunting season



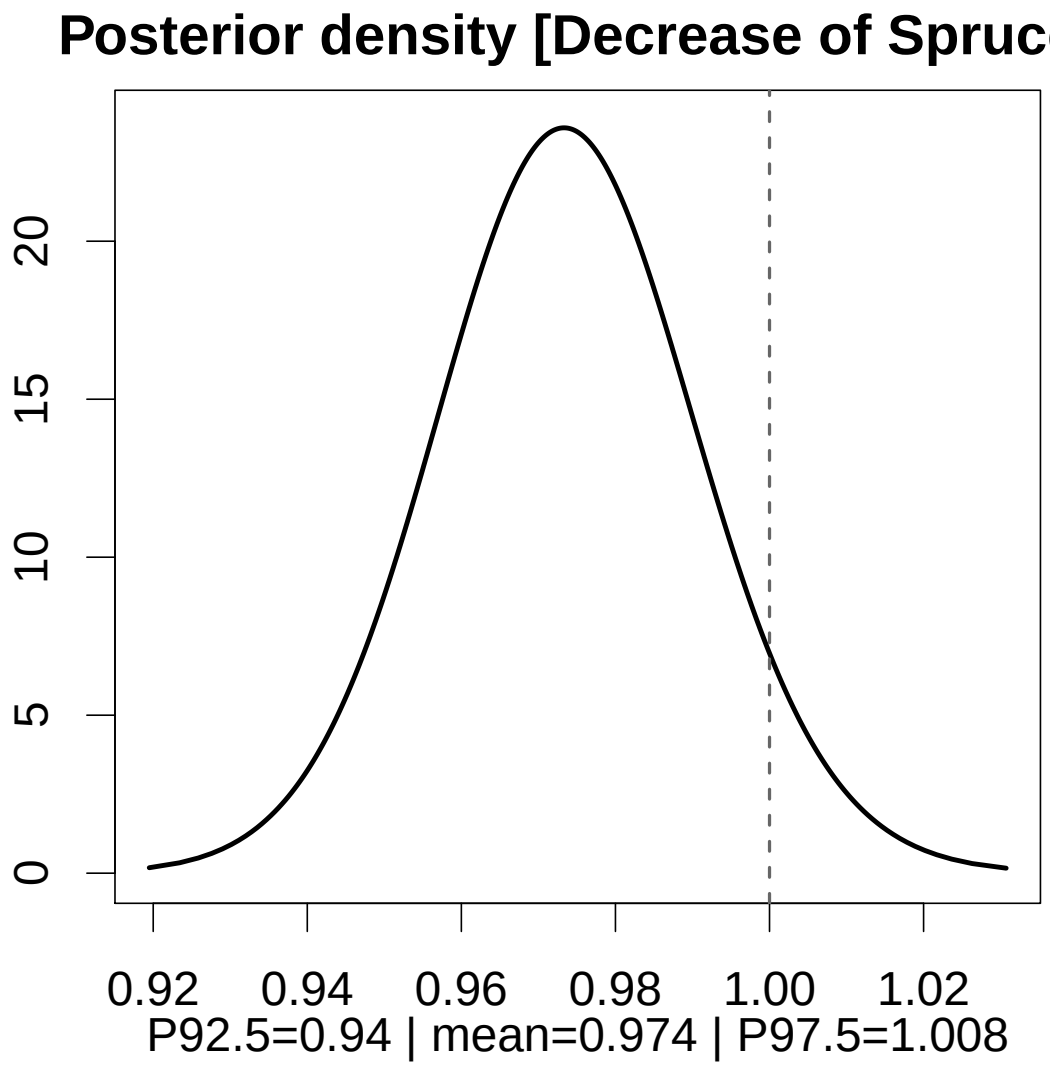
# Fixed linear trend for temperature anomaly



# Fixed linear trend for precipitation



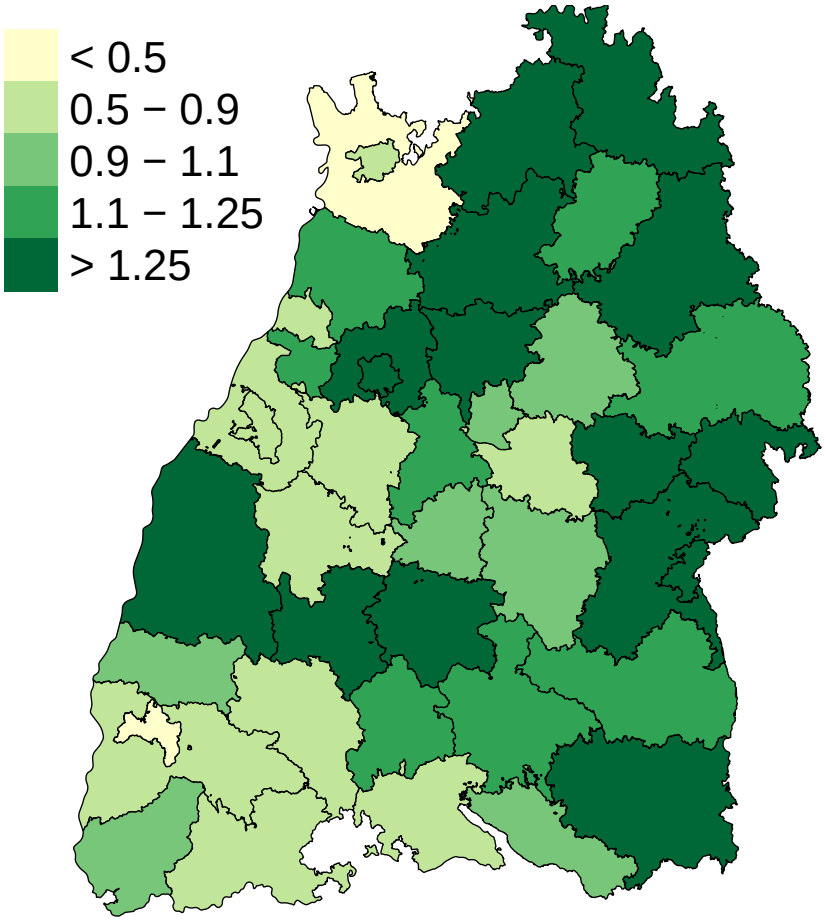
# Fixed linear trend for Spruce decrease



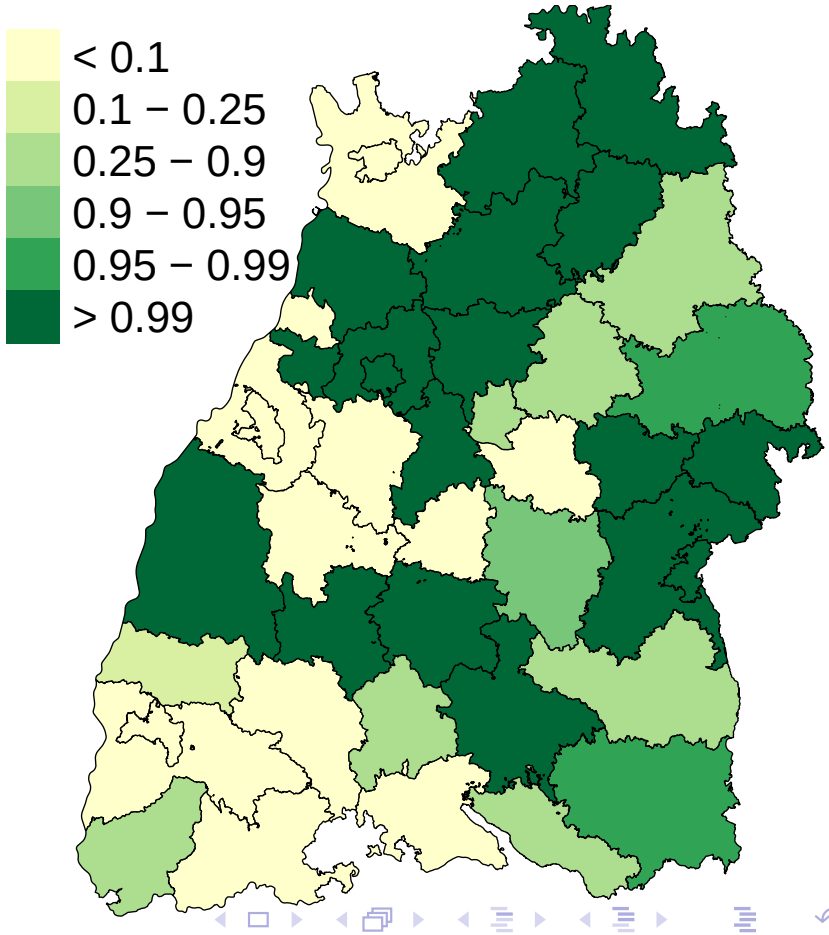
# Spatial effects

Posterior mean of the spatial effect

$$\zeta_i = \exp(\upsilon_i)$$




$$p(\zeta_i > 1 | \boldsymbol{y})$$



# Future work and questions

- Temporal changes of the spatially structured error.
- Varying weekday effect along the year.
- Random walks to model possible nonlinear climatic effects.
- Examine whether decline in 2010 was due to a mast year.
- How does `spBayes` perform?

# The ending

Thank you very much for your attention!